CAPACITY ESTIMATION OF SIMO SYSTEMS IN THE PRESENCE OF CO-CHANNEL INTERFERENCE

Adnan Ali Khan,* and Shahrukh Khalid**

ABSTRACT

This paper presents a new approach for evaluating the performance of single-input multiple-output (SIMO) cellular mobile communication systems in interference limited environment. The performance is analyzed by calculating the ergodic capacity and hence the spectral efficiency for Rayleigh fading SIMO channel in the presence of unequal power Rayleigh fading co-channel interferes. Expressions for evaluating system's capacity using maximum ratio combining (MRC) are presented. The analytical results are verified by developing Monte Carlo simulations. Finally, the effect of variation in cellular parameters including cell size and re-use distance on the system's performance is studied. It is shown that the employment of multiple antennas at the receiver can significantly improve the performance of the system. The capacity limits defined by MRC analysis are found within the estimated values of capacity using simulation. It is observed that increasing cell size and re-use distance decrease the effect of co-channel interference, however, the overall area spectral efficiency (ASE) of the system reduces due to the increase in cellular coverage area. Thus, a trade-off between capacity improvement and reduction in cell-size is suggested.

Key words: *SIMO systems, maximum ratio combining. Monte Carlo simulations, Rayleigh fading, co-channel interference*

INTRODUCTION

The potential of remarkable enhancement in the capacity of wireless systems has attracted considerable interest in multiple-input multiple-output (MIMO) communications. The initial research by Winters[1], Foschini[2], and Telatar[3] incited great attention in this field by envisaging extraordinary spectral efficiency for wireless networks with multiple antennas in which the channel represents rich scattering. MIMO techniques use multiple antennas at transmit and receive sides of a communication network to significantly increase the capacity of system in comparison to conventional single-input single-output (SISO) channel[4]. The method employs spatial diversity technique based on development of information from several signals transmitted over independent fading paths. Its objective is to combine the multiple signals in such a fashion so as to reduce the effects of excessive deep fades.

In the last five years, there has been a wide range of research activities performed for exact calculation of ergodic capacity of SIMO channels. However, the evaluation of system's capacity in the presence of unequal power Rayleigh fading interferers has always remains a challenge due to the complicated distribution of the capacity. In this paper, we worked out capacity of single-input multiple-output (SIMO) systems employing space diversity approach. Expressions for ergodic capacity of the system are derived for maximal ratio combining (MRC) using the fundamental theories presented in[5-7]. In Section I, the system model is introduced and ergodic capacity through analytical approach has been evolved. Section II describes procedure to calculate system's capacity and spectral efficiency through Monte Carlo simulations. Then in Section III, analytical and simulation results are compared and effect of frequency re-use distance and cell size on spectral efficiency is evaluated. Finally, Section V gives the conclusion.

CAPACITY ESTIMATION USING ANALYSIS

A. Analytical Model

We consider a microcellular mobile radio SIMO system built around a channel in flat fading environment having M receiving antennas. We assume that

* *Department of Electrical & Electronic Engineering, The University of Manchester, Manchester M60 1 QD, United Kingdom*

** *Graduate School of Engineering Sciences & Information Technology GSESIT, Hamdard University, Karachi, Pakistan*

receiver antenna array has perfect synchronization so that the receiver can employ coherent detection, and assume that there are k rayleigh faded co-channel interferers. We suppose that the signal of interest and k^{th} interferers are narrowband and experiencing variable flat fading.

The $M \times 1$ discrete-time equivalent received vector takes the form[6]

$$r = \sqrt{\Gamma_0}x_0 g_0 + \sum_{k=1}^{K}\sqrt{\Gamma_k}x_k g_k + n \quad (1)$$

where x_0 and x_k represent transmitted signals of the user of interest and k^{th} interferes correspondingly, which are supposed to be independent and identically distributed (i.i.d.) complex Gaussian vectors with zero mean and unit variance. Γ_0 and Γ_k are short term mean signal strength of the user of interest and the k^{th} interferer. n is the zero average complex AWGN vector, $E[nn^*] = \sigma^2 I_M$, where * represents the transpose conjugate. The vectors h and g_k denote the $M \times 1$ complex channel gains for the user of interest and the k^{th} interferer. We assume that the entries of g_0 and g_k are complex Gaussian variables having zero mean and covariance matrices $r = \sqrt{\Gamma_0}x_0 g_0 + \sum_{k=1}^{K}\sqrt{\Gamma_k}x_k g_k + n$ and $E[g_k g_k^*] = I_M$ respectively.

In order to assess the performance of system, we consider a downlink transmission in a micro-cellular system with dual diversity scheme. We consider the effect of 6 co-channels Rayleigh interferers, located at $D - R$, D and $D + R$, with R being cell radius and D is the reuse distance, as shown in Figure 1[8]. Thus, the interferers average signal power can be calculated as $(D-R)^{-\beta}$, $(D)^{-\beta}$ and $(D+R)^{-\beta}$.

B. *System's Capacity*

With the help of the basic complex model of channel at hand, we can now focus on the main area of concern, i.e. the capacity estimation of SIMO network. The signal-to-interference plus noise ratio (SINR) of the communication link takes the following form

$$SINR = \Gamma_0 \frac{w^* g_0 g_0^* w}{w^*\left(\sigma^2 I_M + \sum_{k=1}^{K}\Gamma_k g_k g_K^*\right)w} \quad (2)$$

C. *Analysis of MRC*

MRC is the combining technique in which the received signals at respective antennas are weighted by a corresponding complex channel gain, i.e. every signal is weighted as per the signal strength. Assuming perfect channel estimation, the weight in case of maximum ratio combining can be stated as $w = g_0^*$. Furthermore, it can be shown that when $g_k \sim CN(O, I_M)(\forall k = 1, 2...., K)$ then Equation (2) can be written as:

$$SINR_{MRC} = \Gamma_0 \frac{g_0^* g_0}{\sigma^2 + \sum_{k=1}^{K}\Gamma_k a_k} \quad (3)$$

$$\text{or } SINR_{MRC} = \Gamma_0 \frac{\sum_{i=1}^{M}|g_i|^2}{\sigma^2 + \sum_{k=1}^{K}\Gamma_k a_k} \quad (4)$$

where $\alpha 1$, $\alpha 2$, $\alpha 3$, ..., αk are independent and exponentially distributed random variables with unit mean.

The capacity of a channel in the flat fading environment can be given according to Shannon's formula[9]:

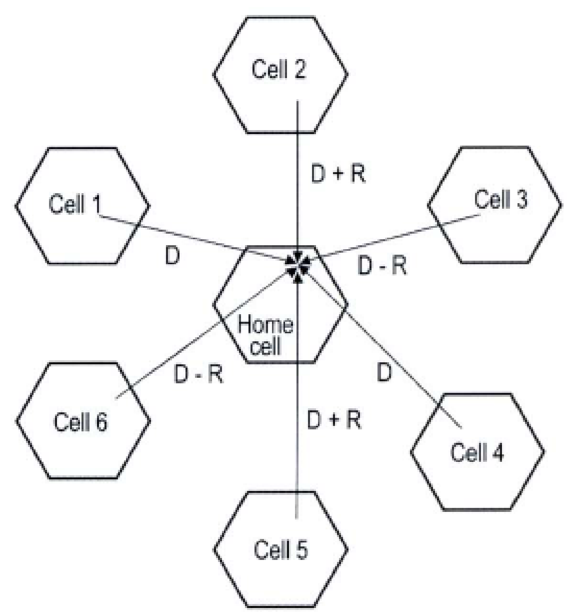

Figure 1: System's model in the presence of six co-channels interferers of the 1[st] tier[8].

$$C = W \log_2(1+SINR) \quad (5)$$

where W is the channel bandwidth. The ergodic capacity is obtained by the normalized average

$$\overline{C}_{MRC} = \frac{E[C]}{W \log_2 e} = \int In(1+x) f_{SINR}(x)dx \quad (6)$$

which presents the theoretical upper bound on spectral efficiency that can be achieved with large block lengths given by,

$$\overline{C}_{MRC} = E\left[\log_2\left(1+\Gamma_0 \frac{\sum_{i=1}^{M}|g_i|^2}{\sigma^2 + K\sum_{k=1}^{K}\Gamma_k a_k}\right)\right] \quad (7)$$

Where the expectation has been obtained over the set of M+2K random variables $\{x_{1,} ..., x_{k,} \Gamma_{1,} ..., \Gamma_k\}$.

Brute-force evaluation of the average in Equation (7) requires a huge computational effort. Here, we present a non-direct approach which leads to a simple explicit expression. It is based on the following result. For any $x,\ y > 0$ we have Equation (6)[10]

$$E\left[In\left(1+\frac{x}{y}\right)\right] = \int_0^o \frac{1}{z}\left(1-e^{-zx}\right)e^{-zy}\, dz \quad (8)$$

Straightforward application of Equation (8) into Equation (7) result in the following desirable result

$$\overline{C}_{MRC} = \int_0^o \frac{1}{z}\left(1-E\left[e^{-z\Gamma_0 g_0 g_0^*}\right]\right)E\left[e^{-z\sum_{k=1}^{K}\Gamma_k a_k}\right]e^{-z\sigma^2 dz} \quad (9)$$

where random variables appears in exponents. This facilitates taking the expectations in terms of the well known results of moments generating function.

Relying on the fact that a_k's are independent and exponentially distributed random variables, results in the following simple expression for the spectral efficiency

$$\overline{C}_{MRC} = \int_0^o \frac{1}{z}\left(1-\frac{1}{\left(1+\Gamma_0 z\right)^M}\right)M\left(z\right)e^{-z\sigma^2}dz \quad (10)$$

where

$$M(z) = E\left[e^{-z\sum_{k=1}^{K}\Gamma_k a_k}\right] = \prod_{k=1}^{K}\frac{1}{1+\Gamma_k z} \quad (11)$$

Equation (10) can be written in the following more convenient form

$$\overline{C}_{MRC} = \int_0^o \frac{1}{z}\left(1-\frac{1}{\left(1+\Gamma_0 z\right)^M}\right)M\left(\frac{1}{\Gamma_0}z\right)e^{-z/SNR}dz \quad (12)$$

where $SNR = \Gamma_0/\sigma^2$ is the signal-to-noise ratio at a single diversity branch in the absence of interference.

For our model, the closed form integral of the system's capacity can be given in Equation (13).

$$\overline{C}_{MRC} = \int_0^o \frac{1}{z}\left(1-\frac{1}{\left(1+\Gamma_0 z\right)^M}\right)\cdot\left(\frac{1}{1+(D)^{-\beta}\ z/\Gamma_0}\right)^2 \cdot\left(\frac{1}{1+(D-R)^{-\beta}\ z/\Gamma_0}\right)^2\cdot\left(\frac{1}{1+(D-R)^{-\beta}\ z/\Gamma_0}\right)^2 e^{-z/SNR}dz \quad (13)$$

CAPACITY ESTIMATION USING SIMULATIONS

The Monte Carlo simulations are developed for validating the analytical results. We assume that the shape of cell is considered as a circle with radius R and frequency-reuse distance is D. The user of interest and co-channel interferers are assumed as mutually independent and uniformly distributed. The PDF of the distance of an arbitrary user is given by:

$$f(r) = \begin{cases} \frac{2r}{R^2} & 0 < r < R \\ 0 & Otherwise \end{cases} \quad (14)$$

The user's position r is calculated using $\Gamma = R\sqrt{u}$ where u is the randomly generated pseudorandom number uniformly distributed in [0, 1].

We consider the multi-cell case with the effect of co-channel interference from the users of the first tier. We assume that the impact of other tiers is negligible as compared to the interference from the first tier cells. The first tier consists of six co-channel cells whose location is described in the model shown is Figure 2.

The polar coordinates of k^{th} co-channel interferer are given by:

$$r_k = R\sqrt{u_k}$$
$$\theta_k = 2\pi v_k \quad (15)$$

Where u_k and v_k are randomly generated pseudorandom numbers distributed in [0, 1].

The distance of the k^{th} interferer from the user Base-station is evaluated using the model described in Figure 2. The distance is given by:

$$d_k = \sqrt{D^2 + r_k^2 + Dr_k \sin(\theta_k)} \quad (16)$$

By the help of Monte Carlo simulation, the SINR and normalized ergodic capacity of the system is evaluated using Equations (4) and (7) respectively. The simulation process is repeated for 10000 iterations and the average capacity of the system is estimated by finding the mean of all values obtained. After 10000 iterations, the value of Capacity $\overline{C}$ converges to three digit accuracy. The average value of capacity $\overline{C}_{mean}$ can be calculated as follows:

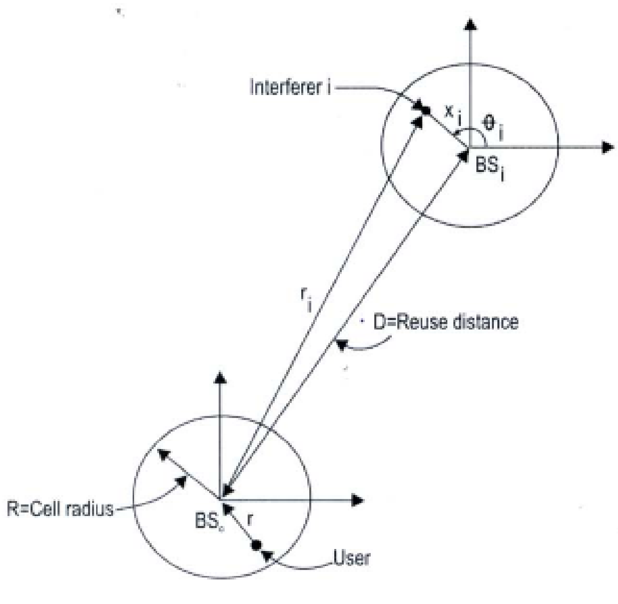

Figure 2: Location of co-channel interferers

$$\overline{C}_{mean} = \frac{\sum_{i=1}^{no.\,of\ iterations} \overline{C}}{No.\,of\ iterations} \quad (17)$$

Once average value of capacity is obtained, the area spectral efficiency (ASE) is calculated using Equation (18).

$$ASE = \frac{4}{\pi R_u^2 R^2} \times \overline{C}_{mean} \quad (18)$$

ANALYTICAL AND SIMULATION RESULTS

A. *Evaluation of capacity of MRC*

This section comprises of comparison and discussion of results obtained by analytical and simulation approaches. For the purpose of analytical similarity, following parameters are kept same for both the analytical and simulation methodology:

1) Path loss exponent $\beta = 4$
2) Cluster size $N = 3$
3) Signal-to-noise ratio SNR = 20dB
4) Number of iteration = 10000

The ergodic capacity of the system estimated against the quantity of receiver's antennas considering MRC is shown in Figure 3. Similarly, the area spectral efficiency of the SIMO link is evaluated w.r.t. number of receiver antennas and the results are depicted in Figure 4.

It is observed that the analytical and simulation results are in very close agreement, which proves the reliability of the presented models. It is noticed that if the number of receiver antennas are kept on increasing, there is a small variation in the capacity of the system. The reason for this result is the fact that the MRC scheme makes best use of channel fading for signal enrichment for every receiver diversity branch. This process of signal enrichment, if applied using large number of multiple antenna, may not result in significantly high variation in capacity as the fading of channel has already been exploited to a very large extent.

It is noticed that SIMO channel is less immune to co-channel interference phenomenon as compared to the conventional SISO system. By utilizing the

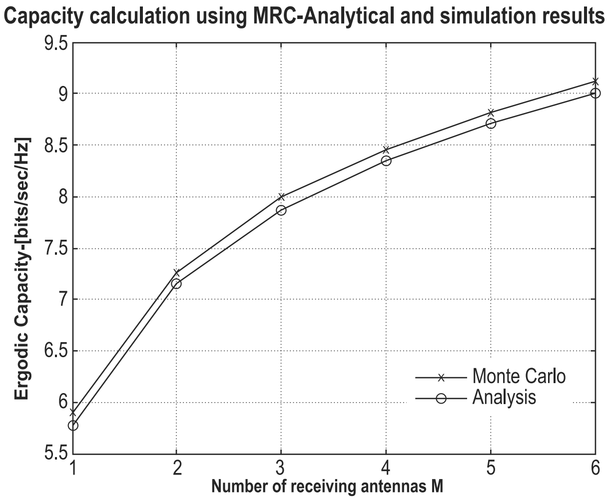

Figure 3: Comparison of ergodic capacity of SIMO system versus number of receiver diversity branches calculated using MRC analysis and simulation. (Fully loaded system with SNR = 20dB)

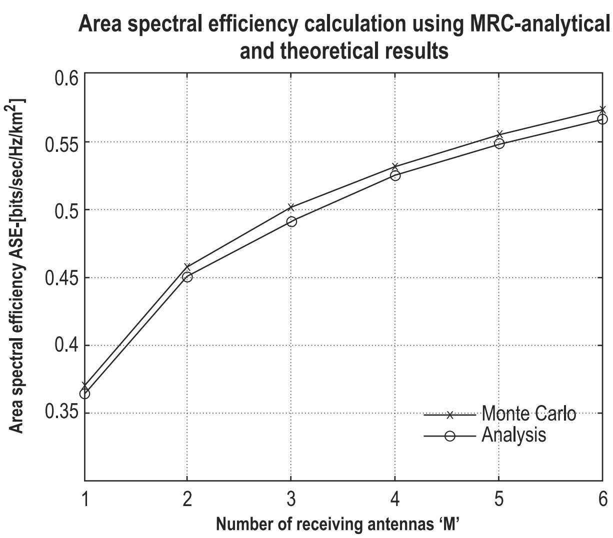

Figure 4: Area spectral efficiency of SIMO system evaluated using MRC analysis and simulation approach for different number of receiver diversity

multiple receive antenna system, the effect of co-channel interference is reduced and performance of the channel is improved.

Figure 4 depicts the area spectral efficiency of SIMO system evaluated using MRC approach for different number of receiver diversity branches. As the capacity increases with the number of receiver antennas, so does the spectral efficiency of the system.

B. Effect of frequency re-use distance on spectral efficiency

In the cellular communication, the re-use of frequency plays a significant role in calculating the efficiency of communication link. In this simulation environment, we test the area spectral efficiency of the SIMO channel with the increase in frequency re-use distance. At first, we plot the area spectral efficiency of the system with respect to the number of receive antennas for different value of re-use distances. This gives us the general trend of the variation in capacity using multiple antennas with re-use distance. The results are shown in Figure 5.

Furthermore, the spectral efficiency of the systems is found with respect to different re-use distances for multiple antenna links of M = 2, 4 and 6. Figure 6 shows the results. It is apparent form Figure 5 and Figure 6 that the Area Spectral Efficiency decreases as an exponential of second-order relative to the re-use distance. Although increasing the reuse distance reduce the effect of co-channel interference, however, it increases the mobile cellular coverage area resulting in an overall decrease of area spectral efficiency of the system.

C. Effect of cell size on SIMO spectral efficiency

This simulation environment is created to observe the effect of cell size on the efficiency of SIMO

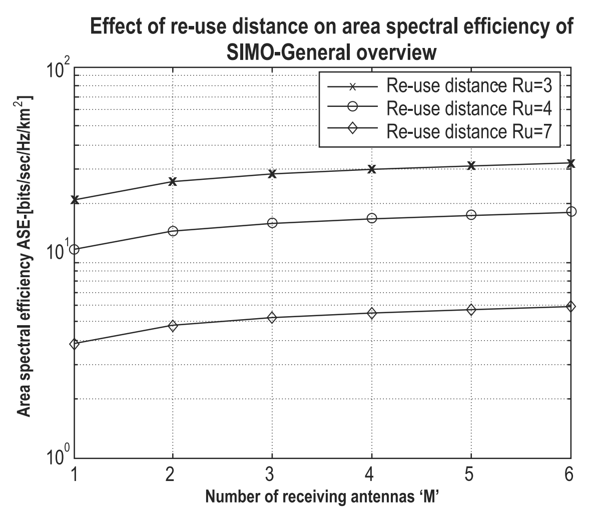

Figure 5: Comparison of area spectral efficiency of SIMO system versus number of receiver diversity branches calculated using different values of re-use distance. (Fully loaded system with SNR = 20dB, cell radius R = 200m, path loss exponent $\beta = 4$)

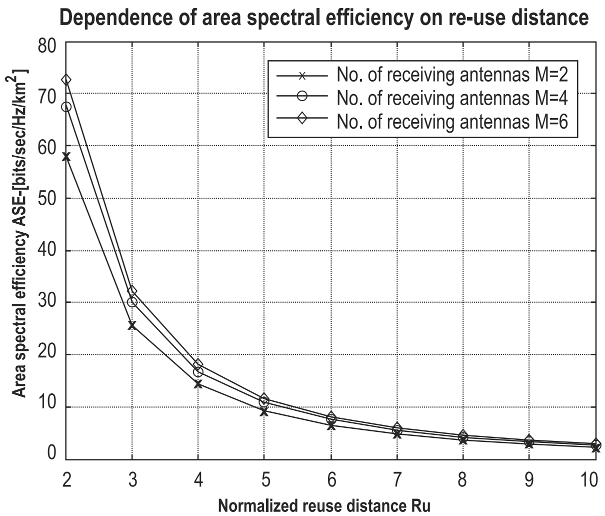

Figure 6: Comparison of area spectral efficiency of SIMO system w.r.t frequency re-use distance for different number of receive antennas M = 2, 4, 6. (Fully loaded system with SNR = 20dB, cell radius R = 200m, path loss exponent β = 4)

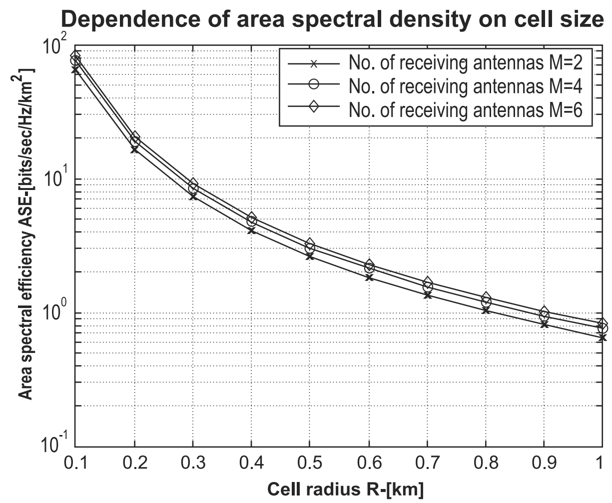

Figure 8: Comparison of the area spectral efficiency of SIMO system versus cell radius for different number of diversity receive antennas M = 2, 4, 6. (Fully loaded system with SNR = 20dB, normalized re-use distance Ru = 4, path loss exponent β = 4)

channel. We have considered the channel with path loss in the first tier of base-station, while interference from other tiers is assumed negligible. The ASE of the system is evaluated for different number of receiver antennas and for different cell sizes. The results in generalized form are shown in Figure 7.

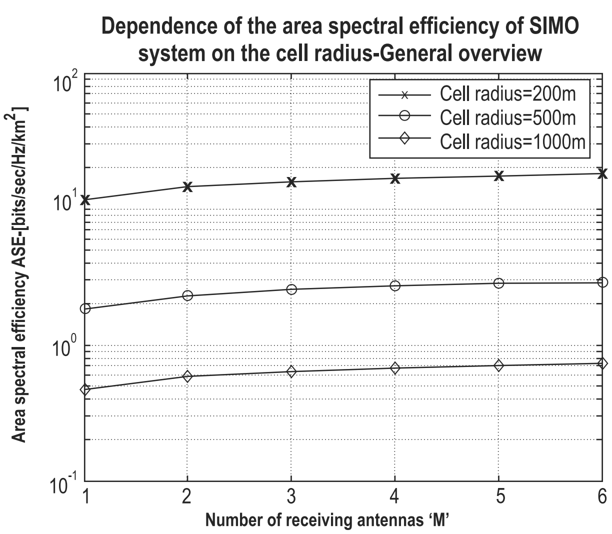

Figure 7: The area spectral efficiency of SIMO system versus number of receive antennas fir different values of cell radii R = 100, 500,1000m. (Fully loaded system with SNR = 20dB, normalized re-use distance Ru = 4, path loss exponent β = 4)

In order to illustrate the effect of cell size more clearly, we plot the ASE w.r.t. the cell size for different number of receiver antennas as depicted in Figure 8.

The ASE of the SIMO channel depends strongly on the cell radius. As the cell size is increased, the ASE falls down rapidly. We can observe that the area spectral efficiency is a function of inverse square of the cell radius which in accordance of our theoretical analysis.

It may be noticed that decreasing the cell size would create more cells and thereby the overall system capacity is increased. However, the effect of co-channel interference is more pronounced with smaller cell size. With larger cell radius, the effect of co-channel interference can be avoided; on the other hand, it increases the mobile coverage area, decreasing the efficiency of system. An optimum cell radius will be the one which gives better spectral efficiency with less immunity to co-channel interference.

CONCLUSION

In this paper, a new methodology for the evaluation of capacity of single-input multiple-output (SIMO) system has been introduced. Considering the effect of co-channel interference, an expression for estimating the ergodic capacity of Rayleigh fading SIMO cellular system has been developed for maximum ratio combining (MRC). The effect of network parameters, like cell size and re-use distance, on the performance of the system is addressed and analyzed in terms of area spectral efficiency (ASE). In addition, Monte Carlo simulations have been used to evaluate the system's capacity and to validate the theoretical analysis.

The results indicate that system's capacity and hence the spectral efficiency of cellular network improve significantly by employing multiple antennas at the receiver of the system.

SIMO system exploits the spatial characteristics of the Rayleigh fading channel which is taken as an environmental source of possible signal enrichment. Hence, it gives rise to a significant enhancement in the channel capacity by receiving multiple copies of transmitted signal. The bounds on capacity are found well within values of simulated results. It is noticed that the SIMO system is less vulnerable to the co-channel interference problem as compared to the to the traditional single- input single-output (SISO) channel.

Furthermore, the work also explored the analysis of the effect of cell size and re-uses distance on the area spectral efficiency of SIMO link. It is noticed that decreasing the cell size and re-use distance create more cells and thereby the overall system capacity is increased.

REFERENCES

1. Winters, J. 1987. On the capacity of radio communication systems with diversity in a Rayleigh fading environment, IEEE J. Select. Areas Commun., vol. 5, 871-878.
2. Foschini, G.J. 1996. Layered space-time architecture for wireless communication in fading environments when using multi-element antennas, Bell Labs Tech. J., 41-59.
3. Telatar, E. 1999. Capacity of multi-antenna Gaussian channels, Eur Trans.Telecomm. ETT, vol. 10, no. 6, 585–596.
4. Catreux, S., Driessen, P.F. and Greenstein, L.J. 2002. Data throughput using multiple-input multiple-output (MIMO) techniques in a noise limited cellular environment, IEEE trans. on commun., vol. 1, no. 2, 1.
5. Cui, J., Falconer D.D. and Sheikh, A.U.H. 1998. Performance evaluation of optimum combining and maximal ratio combining in the presence of co-channel interference and channel correlation for wireless communication systems, Mobile Networks and Applications archive, vol. 2(4).
6. Zhao, Y., Zhao, M., Zhou, S. and Wang, J. 2005. Capacity bounds of SIMO systems with co-channel interferers, IEICE Trans Fundamentals, vol. E88-A, no. 8, 2231-2235.
7. Gao, H. and Smith, P. 1998. Some bounds on the distribution of certain quadratic forms in normal random variable, Austral. & New Zealand J. Statist. vol. 40, no. 1, 73-81.
8. Agrawal, D.P. and Zeng, Q. 2002. Introduction to Wireless and Mobile System. University of Cincinnati.
9. Proakis, J.G. 1989. Digital Communications. New York: McGraw-Hill.
10. Hamdi, K.A. 2008. Capacity of MRC on correlated Rician fading channels, IEEE trans. on commun, vol. 56, pp. 708-711.