

## A TUTORIAL ON ADAPTIVE ROBUST POSITION CONTROL OF DC MOTOR USING FUZZY LOGIC COMPENSATION

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### ABSTRACT

*Abstract.* This article presents a tutorial on adaptive fuzzy sliding mode control for position control of DC servo motor which is subjected to nonlinear friction and external disturbance torque. Friction phenomenon is simulated using LuGre dynamic model. Adaptive fuzzy system is used to approximate nonlinear friction and external disturbance. Assuming that system state matrixes are known, model based robust controller is formulated using sliding mode control method. Adaptive fuzzy system formulated using Lyapunov function. The proposed controller is derived for DC motor position control system and numerical simulations are presented to verify its effectiveness.

**Keywords:** Adaptive robust tracking control, DC motor, LuGre dynamic friction, Disturbance torque, fuzzy logic.

### INTRODUCTION

In high precision position control systems, nonlinear friction and external disturbances have a negative effect on control accuracy. Nonlinear friction is compensated using model based empirical relations which are not true representation of the actual phenomenon. It can be very prominent at low velocities. Simple PID control method is not very effective in presence of nonlinear factors and unknown disturbances. It is hard to compensate nonlinear phenomenon's using model based compensation methods. Adaptive fuzzy system is proposed to compensate nonlinearities of robot manipulators and experimental validation is provided<sup>1</sup>.

Model based approach is proposed to compensate friction phenomenon using dynamic friction model<sup>2,3</sup>. According to universal approximation theorem, Fuzzy systems are used to approximate any nonlinear continuous function<sup>4</sup>. Classical sliding mode control effectively compensates nonlinearities and disturbances with known upper bounds but it induces high frequency chattering which is not feasible for implementation point of view. To preserve robustness and rectify chattering problem in sliding mode control method, nonlinearities and external disturbances should be estimated online. Most common approach is observer based sliding mode control but it is a model based approach. Major limitation of model based approach is that system matrixes should be exactly known. Model free approach is fuzzy sliding mode control method in which adaptive fuzzy system is used to estimated nonlinearities online<sup>5-7</sup>. Adaptive fuzzy system with classical sliding mode control reduces gain of

discontinuous control term, thus chattering phenomenon is minimized while robustness property is still preserved.

Based on the above literature survey this article introduces a tutorial on adaptive robust fuzzy sliding mode control for DC motor position control with uncertainties which is subjected to friction and unknown disturbance torque. Nonlinear model of DC motor is derived taking effect of LuGre friction model and load disturbance and then based on the derived non linear model the adaptive robust control law is formulated.

### Basic Definitions

In multi input multi output (MIMO) fuzzy system configuration, output parameter vector is represented as.

$$y_j = \frac{\sum_{l=1}^M \left( \prod_{i=1}^n u_{A_i^l}(x_i) \right) y_j^{-l}}{\sum_{l=1}^M \left( \prod_{i=1}^n u_{A_i^l}(x_i) \right)} \quad j = 1, 2, \dots, m \quad (1)$$

Equation (1) is simplified as

$$y_j = \sum_{l=1}^M y_j^{-l} \xi(x) \quad j = 1, 2, \dots, m \quad (2)$$

$$y_j = \theta_j \xi(x) \quad j = 1, 2, \dots, m \quad (3)$$

Here  $\theta_j$  represents parameter vector and  $\xi(x)$  is fuzzy bases function vector which is represented as;

$$\xi(x) = \frac{\left( \prod_{i=1}^n u_{A_i^l}(x_i) \right)}{\sum_{l=1}^M \left( \prod_{i=1}^n u_{A_i^l}(x_i) \right)} \quad l = 1, 2, \dots, M \quad (4)$$

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### System Model

In this section DC motor dynamic model is discussed in details including the effect of nonlinear friction and external disturbances. Mechanical and electrical dynamics are represented as

$$U = Ri_a + L \frac{di_a}{dt} + K_b \dot{\theta} \quad (5)$$

$$T_e = J\ddot{\theta} + \beta\dot{\theta} + T_L + T_f \quad (6)$$

Here  $T_e = K_m i_a$ ,  $[T_L, T_f]$  represents external load torque and friction torques respectively.  $[R, L]$  represent electrical resistance and winding inductance,  $[J, \beta]$  are rotational inertia and damping coefficient,  $[U, i_a]$  represents electrical voltage and armature current and  $[K_m, K_b]$  represents motor torque constant and back emf constant. From Equation. 5 and Equation. 6, state equations can be written as

$$\dot{x}_1 = x_2 \quad (7)$$

$$\dot{x}_2 = -ax_2 + bu - cT_L - dT_f \quad (8)$$

Here  $[x_1, x_2]$  represents angular position and velocity of motor and  $[a, b, c]$  represents system matrixes. Luge friction mode is formulated as

$$T_f = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v \quad (9)$$

$$\dot{z} = v - \frac{\sigma_0 |v|}{g(v)} \quad (10)$$

$$g(v) = f_c + (f_c - f_s) e^{-\left(\frac{v}{v_s}\right)^2} \quad (11)$$

### Control Law Formulations

Let  $\theta$  be ideal position signal;  $\theta_d$  be the desired angle command. Error vector is;

$$e = \theta - \theta_d \quad (12)$$

$$\dot{e} = \dot{\theta} - \dot{\theta}_d \quad (13)$$

Sliding surface is;

$$s = \dot{e} + \lambda e \quad (14)$$

$$\dot{s} = \ddot{e} + \lambda \dot{e} \quad (15)$$

A new parameter is defined as;

$$\dot{\theta}_r = \dot{\theta}_d - \lambda e \quad (16)$$

$$\dot{\theta}_d - \dot{\theta}_r = \lambda e \quad (17)$$

Combine Eq. 13, Eq. 14 and Eq. 17

$$s = \dot{\theta} - \dot{\theta}_r \quad (18)$$

$$\dot{s} = \ddot{\theta} - \ddot{\theta}_r \quad (19)$$

Combine Eq. 8 and Eq. 19. After simplifying;

$$\dot{s} = -ax_2 + bu - f(T_L, T_f) - \ddot{\theta}_r \quad (20)$$

$$u = \frac{1}{b}(ax_2 + \hat{f}(T_L, T_f|\theta) + \ddot{\theta}_r - K_d s - wsgn(s)) \quad (21)$$

Here  $\hat{f}(T_L, T_f|\theta)$  is estimated friction and disturbance torque using fuzzy logic system. Reaching law is selected as  $\dot{s} = K_d s - wsgn(s)$  and  $[K_d \ w]$  represents reaching law gain. Figure 1 shows block diagram of the proposed control scheme.

To prove stability of the closed loop system, Lyapunov function is written as;

$$V = \frac{1}{2}(s^2 + \sum_{i=1}^n \eta_i \tilde{\theta}_i^2) \quad (22)$$

Here  $\tilde{\theta}_i = \tilde{\theta}_i - \theta$ . Differentiating Eq. 22;

$$\dot{V} = s\dot{s} + \sum_{i=1}^n \eta_i \tilde{\theta}_i \dot{\tilde{\theta}}_i \quad (23)$$

Combine Eq. 20 and Eq. 23 results in

$$\dot{V} = s(-ax_2 + bu - f(T_L, T_f) - \ddot{\theta}_r) + \sum_{i=1}^n \eta_i \tilde{\theta}_i \dot{\tilde{\theta}}_i \quad (24)$$

Define fuzzy approximation error

as  $e_f = f(T_L, T_f) - \tilde{f}(T_L, T_f|\theta^*)$ . Eq. 24 is written as

$$\dot{V} = s(\tilde{f}(T_L, T_f|\theta) - f(T_L, T_f) - K_d s - wsgn(s)) + \sum_{i=1}^n \eta_i \tilde{\theta}_i \dot{\tilde{\theta}}_i \quad (25)$$

$$\dot{V} = s(\tilde{\theta}_i \xi_i(\theta, \dot{\theta}) - e_f - K_d s - wsgn(s)) + \sum_{i=1}^n \eta_i \tilde{\theta}_i \dot{\tilde{\theta}}_i \quad (26)$$

$$\dot{V} = s(-e_f - K_d s - wsgn(s)) + \sum_{i=1}^n \eta_i \tilde{\theta}_i \dot{\tilde{\theta}}_i + s_i \tilde{\theta}_i \xi_i(\theta, \dot{\theta}) \quad (27)$$

Define adaptive law as;

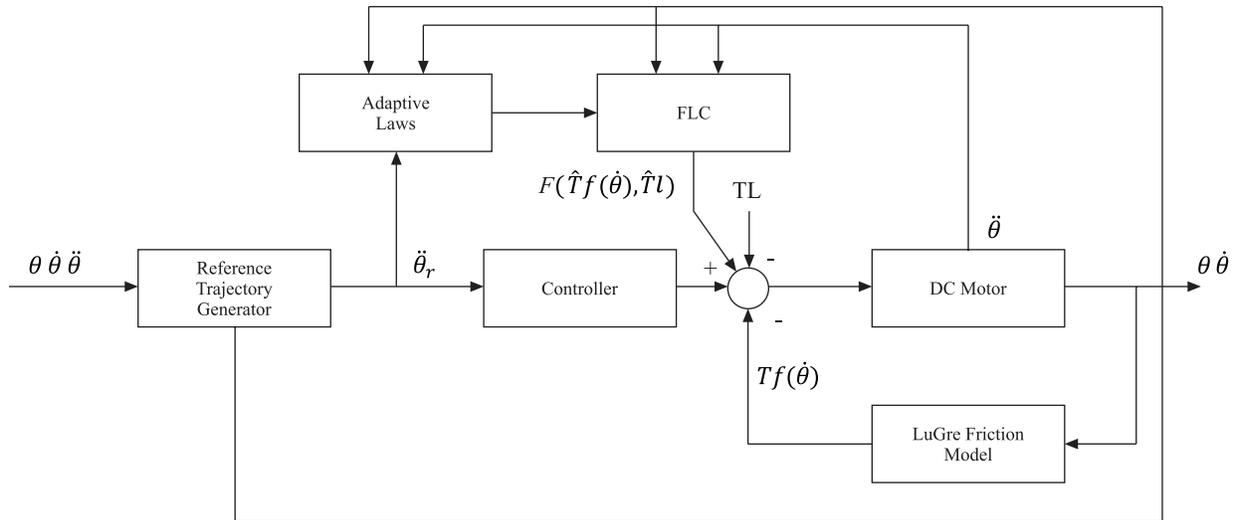


Figure 1: Proposed control scheme

$$\dot{\xi}_i = -\eta_i^{-1} s_i \xi_i(\theta, \dot{\theta}) \tag{28}$$

$$\dot{V} = s(-e_f - K_d s - w \operatorname{sgn}(s)) \tag{29}$$

It is assumed that ideally  $e_f$  is approaching zero. Sliding condition will occur if  $K_d > 0; > \delta_{max}$ . Here  $\delta_{max}$  represents system uncertainty including fuzzy approximation error. Eq. 29 can be written as

$$\dot{V} = -sK_d s - w|s| \leq 0 \tag{30}$$

## RESULTS AND SIMULATIONS

To implement the proposed scheme, choose position ideal signal as  $\theta = 0.05 \sin(2 * \pi * t)$  and external disturbance signal as  $T_d = 10 * (\sin 2 * \pi * t)$ . Parameters of dynamic friction model are selected as follow  $\sigma_0 = 260 \text{ Nm/rad}$ ,  $\sigma_1 = 2.5 \text{ Nm s/rad}$ ,  $\sigma_2 = 2.5 \text{ Nm s/rad}$ ,  $f_c = 2.5 \text{ Nm}$ ,  $f_s = 2.3 \text{ Nm}$ ,  $v_s = 0.01 \text{ rad/s}$ . For controller choose the following parameters.  $K_d = 10$ ,  $w = 1.5$ ,  $\lambda = 20$ ,  $\eta_i = 0.0001$ . Figure 2 shows estimated friction using fuzzy logic system. It is clear that friction estimation

error is very small. Similarly Figure 3 shows unknown disturbance estimation results and from simulations it is clear that estimation error is very small. From Figure 2 and Figure 3, there are overshoots in transient time. These overshoots can be minimized by selecting appropriate learning rate for adaptive fuzzy system.

Figure 4 shows position tracking performance of the proposed control scheme. Tracking error is very small which is shown in Figure 5. Figure 6 shows control signal simulations. Chattering phenomenon is effectively minimized as compared to classical sliding mode control method mentioned in the above cited literature.

## CONCLUSION

A tutorial on adaptive robust control using fuzzy logic compensation is presented in this article. The proposed adaptive fuzzy compensation can be used to estimate unknown disturbance and dynamic friction very accurately. Tracking effectiveness is shown in figure .3 under the influence of friction and unknown disturbance torque.

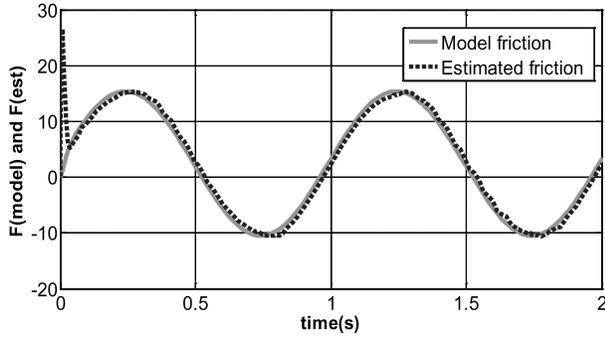


Figure 2: Friction Estimation

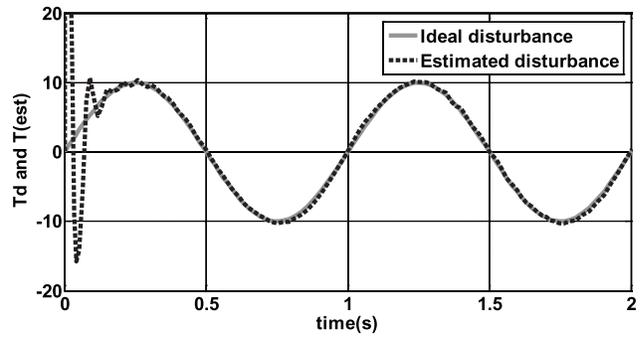


Figure 3: Disturbance Estimation

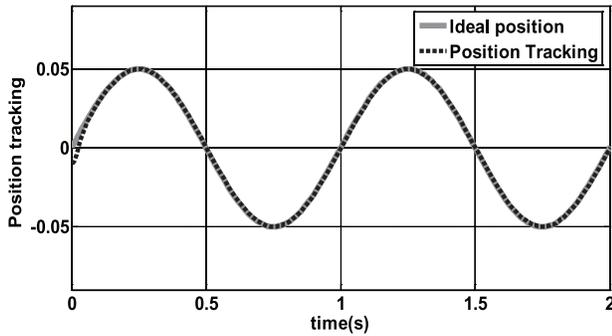


Figure 4: Position Tracking

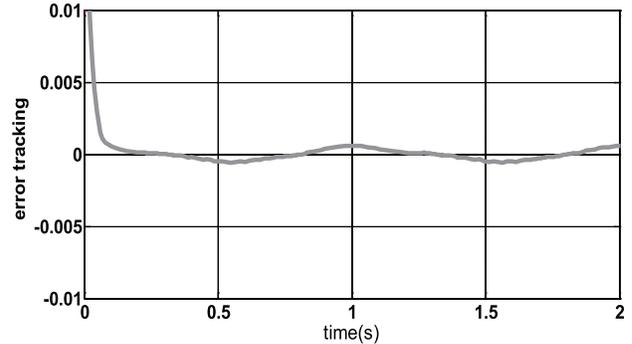


Figure 5: Tracking Error

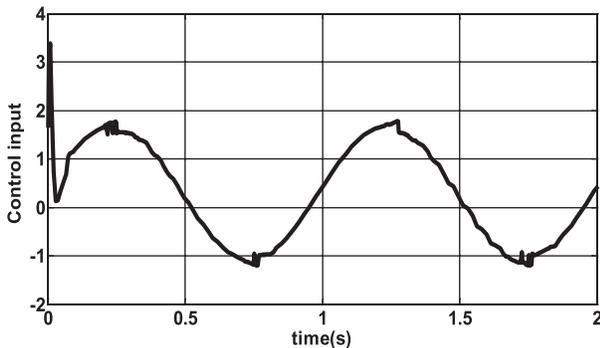


Figure 6: Control Input Simulations

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